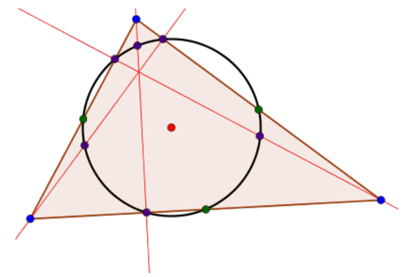
Euclidean Geometry: An Introduction to Mathematical Work

Math 3600

Spring 2017



Circles

We have learned quite a bit about basic polygonal shapes, especially triangles, and various species of quadrilaterals. Now we turn our attention to circles. This is the subject of Book III in Euclid's *Elements*. We already have one beautiful theorem about circles, that of Thales, but we'd like to have more.

Read the *Elements* Book III Propositions 1-34. For the following propositions you should work in the axiomatic style of Euclid using I.1-34, III.1-34 and any previously proved results.

Pay special attention to III.16, III.18, III.20, III.21, III.31 and III.32.

9.1 Conjecture. Let AB and AC be two tangent lines from a point A outside a circle. Then AB is congruent to AC.

Definition. We say that two circles *meet at right angles* if the radii of the two circles to a point of intersection make a right angle.

9.2 Conjecture. Let Γ and Ω be two circles with centers G and O, respectively. Suppose that these circles meet at two points A and B. If GAO is a right angle, then GBO is a right angle.

Definition. A quadrilateral $ABCD$ is said to be a *cyclic quadrilateral* if there is a circle Γ such that the four vertices A, B, C and D lie on Γ.

9.3 Conjecture. A rectangle is always a cyclic quadrilateral.

9.4 Conjecture (Cyclic Quadrilateral Theorem)**.** Let A, B, C and D be four points. The quadrilateral $ABCD$ is cyclic if and only if angle DAC is congruent to DBC.

9.5 Conjecture. Let two circles be tangent at a point A. If two lines are drawn through A meeting one circle at further points B and C and meeting the other circle at points D and E, then BC is parallel to DE.