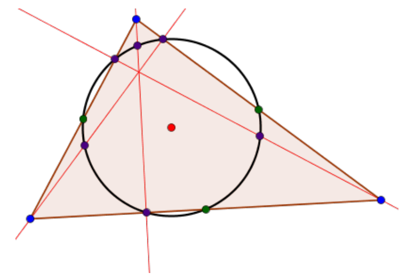
Euclidean Geometry:
An Introduction to Mathematical Work

Math 3600

Spring 2017



The Center of a Triangle

What might be called the center of a triangle? There have been many proposed answers to this question over the centuries. In this assignment, we study two of them.

8.1 Conjecture. Let ABC be a triangle, with rays r and s the angle bisectors at A and B, respectively. Suppose that r and s meet at the point I which lies inside the triangle. Draw lines l and m through I that are perpendicular to AC and BC respectively. If l meets AC at point X and m meets BC at Y, then triangle IXC is congruent to triangle IYC.

Definition. Three segments (or lines or rays) are called *concurrent* if they all pass through a common point.

8.2 Conjecture. The three angle bisectors of a triangle are concurrent.

Definition. The point just discovered is called the *incenter* of the triangle.

8.3 Conjecture. Let T be a triangle. For any pair of sides of T, the perpendicular bisectors of those sides meet. (That is, they are not parallel.)

8.4 Conjecture. The three perpendicular bisectors of any triangle are concurrent.

Definition. The point where the three perpendicular bisectors of a triangle meet is called the *circumcenter* of the triangle.