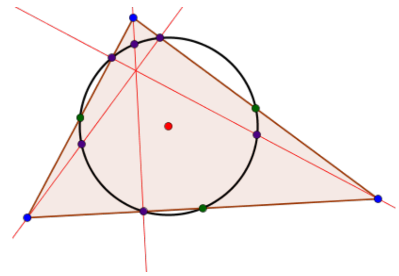
Euclidean Geometry:
An Introduction to Mathematical Work

Math 3600

Spring 2017



The Geometry of Rectangles

Rectangles are probably familiar to you, but to be clear we give a precise definition.

Definition. A *rectangle* is a quadrilateral which has all four interior angles that are right angles.

Notice that the definition only speaks about angles. There is nothing at all said about the sides. But that doesn't mean that the sides have no special properties—it is just that those properties are really theorems.

This is something many people have to get used to. It is a way in which mathematical definitions differ from common definitions in English.

3.1 Conjecture. Let R be a rectangle. Then R is a parallelogram.

3.2 Conjecture. Let R be a rectangle. Then each pair of opposite sides of R is a pair of congruent segments.

3.3 Conjecture. The two diagonals of a rectangle are congruent and bisect each other.

These conjectures describe some qualities that are common to all rectangles. Mathematicians say that these are *necessary conditions* for having a rectangle. If a figure is a rectangle, then these properties are "necessarily" also true. Now we shall try to go in the opposite direction. The following conjectures are possible answers to the question, "What kind of information does one need to claim that that our figure is a rectangle?" Of course, we can use the definition above–that is one of the important roles of a definition, to test when we are allowed to use a word. But what we want now are *sufficient conditions*, these are conditions that allow us to conclude that our figure is a rectangle by checking something other than the definition.

We will later see an example of conditions which are both *necessary* and *sufficient* for something, and thus provide an example of an *equivalent statement*.

3.4 Conjecture. Let $ABCD$ be a quadrilateral such that angles $\angle ABC$ and $\angle ADC$ are right angles. If segments AB and CD are congruent, then $ABCD$ is a rectangle.

3.5 Conjecture. Let $ABCD$ be a quadrilateral such that angles $\angle ABC$ and $\angle ADC$ are right angles. If segments AB and CD are parallel, then $ABCD$ is a rectangle.

Notice the importance that pairs of parallel lines play in these statements.

These results don't *look* to be about rectangles.

3.6 Conjecture (Midline Theorem)**.** Let ABC be a triangle, D the midpoint of AB and E the midpoint of AC. Then the line through E and D, called a *midline*, is parallel to the line through B and C.

3.7 Conjecture (Varignon's Theorem)**.** Let $ABCD$ be a quadrilateral. The midpoints of the four sides are the vertices of a parallelogram.

3.8 Conjecture (Kinda Tricky, version one)**.** Let $ABCD$ be a quadrilateral. If AB is congruent to CD and BC is congruent to AD, then $ABCD$ is a parallelogram.

3.9 Conjecture (Kinda Tricky, version two)**.** Let $ABCD$ be a quadrilateral. If AB is congruent to CD and angle BCD is congruent to DAB, then $ABCD$ is a parallelogram.