# BISHOP PARALLEL FRAMING AND PARALLEL CURVES PROJECT

### DIFFERENTIAL GEOMETRY, SPRING 2015

## Central Theme

The Frenet-Serret framing along a curve only makes sense as long as the curvature  $\kappa$  never vanishes. Points where  $\kappa = 0$  make the normal direction undefined, and thus the binormal, too. To get around this hassle, Richard Bishop proposed a new set-up he called *Relatively Parallel Adapted Frames.* This notion is related to the idea of having two parallel curves.

#### MINIMUM REQUIREMENTS

Write a paper exploring the basics of the Bishop's framings and parallel curves.

- $\bullet$  7-10 pages, in  $\ensuremath{\mathbb{E}}\xspace{TE}\xspace{X}\xspace{X}$  , with attention paid to standard English grammar, spelling and usage.
- Give a clear definition of the Bishop framing.
- Compute several examples of curves for which the Bishop framing might be a necessary thing.
- Compute some examples of curves parallel to a given curve. Include at least a line, a circle, a circular helix, and the twisted cubic (Shifrin page 3).
- Include images where appropriate.
- Solve the following exercises out of Shifrin and weave them into a coherent story: \$1.2#23, \$1.3#9-11.

## EXTENSIONS TO EXPLORE

Read through Bishop's paper and figure out the relationship between the functions  $k_1, k_2$ from a Bishop framing and the standard  $\kappa$  and  $\tau$  from the Frenet-Serret Apparatus.

#### RESOURCES

The original paper by Richard Bishop appeared in the American Mathematical Monthly in 1975, and is available on jstor. If you are on campus, this url will work:

http://www.jstor.org/stable/2319846